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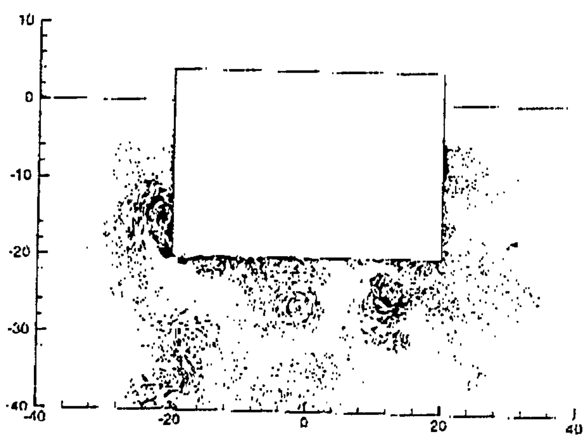

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A New Model on Equilibrium Spectrum of Wind Waves

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ABSTRACT

Based on the joint-distribution of wave amplitude and wave period, a new model on the equilibrium spectrum of wind waves is derived. The derived model is related to the wind speed, spectral width and wave age. The related coefficients are determined empirically based on the observation. This model can be used to explain the variability of spectral level and spectral curve slope. It is found that the spectral level is controlled by both wind speed and wave age; the slope of spectral curve is related to only the wave age. The relationships between the spectral width and wave age, between the spectral level and wave age are obtained based on the spectral properties and empirical observations.

KEY WORDS: waves, spectrum, equilibrium, wave age, spectral width.

1 INTRODUCTION

Generally, the "equilibrium spectrum" is limited to the high frequency (or wavenumber) tail rather than the entire frequency (or wavenumber) range. The first model of equilibrium frequency spectrum for deep water waves was proposed by Phillips (1958) based on dimensional analysis as

$$S(\omega) = \alpha g^2 \omega^{-5}. \tag{1}$$

The corresponding unidirectional wavenumber spectrum is

$$S(k) = \frac{\alpha}{2} k^{-4}, \tag{2}$$

where α is the equilibrium constant, g is the gravitational acceleration, ω is the angular frequency, k is the wavenumber.

For the frequency region $1.5\omega_p < \omega < 3.5\omega_p$ and the wave age region $0.83 < \frac{U_{10}}{C_p} < 5$, Donelan et al. (1985) obtained

$$S(\omega) = 0.006(\frac{U_{10}}{C_p})^{0.55}(\frac{\omega}{\omega_p})\omega^{-5} exp(-\frac{\omega_p^4}{\omega^4})\gamma^{\Gamma} \tag{3}$$

from the observations of water surface elevation using 14 wave staffs in an array in Lake Ontario and in a large laboratory tank. Here ω_p is the angular frequency at the spectral peak, C_p is the phase speed of the waves at spectral peak, γ^{Γ} is the peak enhancement (over the Pierson-Moscowitz spectrum) factor.

In this paper, we will derive the equilibrium spectrum of wind waves, based on both dynamical and statistical considerations. At first, the wind energy is input through the work done by air presure component induced by large-scale water waves. In smaller scale, the wind energy can be directly input through the wind friction stress. The two types of wind input may overlap in a part of the equilibrium range. Therefore, the measured equilibrium spectra of wind waves should include both contributions from the two mechanisms. The equilibrium spectrum derived in this paper represents the energy density contributed by the first mechanism. The spectrum of wind-induced gravity-capillary waves (Liu 1996; Liu and Yan 1995) may represent the energy density contributed by the second mechanism. The sum of the above two spectra may represent the energy density in the equilibrium range, especially for higher frequency range and stronger wind condition. In the statistical consideration, a model of equilibrium spectrum will be derived from the joint-distribution of wave amplitude and wave period of gravity waves. In other words, the model is derived from theories and observations on the statistical property of wave characteristic variables, rather than the direct spectral estimate. In detail, the joint-distribution of amplitude and wave period is given based on the probability density function (PDF) of wave period of Liu et al. (1997), the PDF of wave amplitude given by Longuet-Higgins (1975), and an assumption on the correlation between wave period and wave amplitude. The assumption on the correlation between wave period and wave amplitude is obtained based on the observations of Gooda (1977). The spectral width-related parameter has been included in the model of wave period distribution (Liu et al. 1997). Therefore, the derived spectrum is related to the spectral width. Further, the dependence of spectral width and spectral level on wave age will be obtained based on the spectral property and the observations of other investigators (Donelan et al. 1985; Perrie and Toulany 1990). Because of relation with wave age, the derived model can be used to investigate the influence of wave age on the mean-square slope and further radar reflection at the sea.

2 JOINT-DISTRIBUTION OF WAVE AMPLITUDE AND WAVE PERIOD

A probability density function (PDF) of wave period with a spectral width-related parameter n, suggested by Liu et al. (1997) based on the theory of Longuet-Higgins (1975) and the observations of Davidan et al. (1973) and Bretschneider (1959), is

$$f(T) = \frac{(2n-1)^{n/2}}{\Gamma(\frac{n}{2})2^{n-2}} \frac{T^{2n-1}}{T_m^{2n}} exp[-\frac{(2n-1)T^4}{4T_m^4}], \quad (4)$$

where T is the wave period, T_m is the most-probable wave period, the Γ-function is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx \quad (5)$$

for $\alpha > 0$.

The PDF of the wave amplitude a is given by the Rayleigh distribution (Liu et al. 1997)

$$f(a) = \frac{a}{a_m^2} exp(-\frac{a^2}{2a_m^2}), \quad (6)$$

where a_m is the most-probable amplitude.

The PDF of wave period of Longuet-Higgins (1975) was derived based on an assumption of narrow spectrum. However, Liu et al. (1997) have confirmed that equation (4) can describe the PDF of wave period for both wide spectrum and narrow spectrum. With a larger value of n, equation (4) is in keeping with Longuet-Higgins's model. Also (4) with $n = 2.0$ fits the measurements of Bretschneider (1959) very well; and (4) with $n = 1.25$ fits the measurements of Davidan et al. (1973) very well. Therefore, the parameter n in (4) denotes a spectral width-related parameter. The PDF of wave amplitude of Longuet-Higgins (1975) was derived based on the assumption of narrow spectrum, and it is used only in a narrow frequency subrange in this study. It should be indicated that a numerous measurements (Liu et al. 1997) have confirmed that the PDF of wave amplitude is approximately unrelated to the spectral width. In this study, the joint-distribution of wave amplitude and wave period will be derived based on the PDF of wave period (4) and the PDF of wave amplitude (6).

According to probability theory, the joint-PDF of wave amplitude and wave period can be expressed by

$$f(a,T) = f_1(a/T)f_2(T), \quad (7)$$

where f_2 is the PDF of wave period expressed by (4), $f_1(a/T)$ is the conditional PDF of wave amplitude expressed by (6), in which a_m is the most-probable amplitude in a narrow period subrange dT corresponding to T.

The wave amplitude and the wave period are correlated to each other. A formula to express the correlation is

$$\frac{a_m}{A_m} = (\frac{T}{T_m})^r, \quad (8)$$

where A_m is the most-probable amplitude corresponding to T_m, T_m is the most-probable period, r is the correlation-related parameter. It is found that equations (7) and (8) with $r = 1$ can describe the observations of Gooda (1977) very well for the subrange of $T/T_m < 1$, which just corresponds to the equilibrium range. Because the subrange of $T/T_m < 1$ corresponds to the equilibrium range, we will use $r = 1$ to describe the correlation between the wave period and the wave amplitude for the equilibrium spectrum. We will also use $n = 1.25$ to describe the spectral width for the equilibrium spectrum in fully-developed stage. The value of $n = 1.25$ is obtained based on the observations of Davidan et al. (1973) as described earlier (Liu et al. 1997). Generally, the statistical average corresponds to the fully-developed stage, rather than the developing stage or overdeveloped stage. The overdeveloped waves are those with a peak frequency lower than that of the fully-developed waves. The developing waves have a peak frequency higher than that of the fully-developed waves. Taking together, we have

$$n + r = 2.25, \quad (9)$$

which represents the value of $(n + r)$ suitable for fully-developed waves.

3 EQUILIBRIUM SPECTRUM OF WIND WAVES

a. Derivation of spectrum

Based on probability theory, the spectrum of wind waves can be expressed by

$$S(\omega) = S(T)\frac{dT}{d\omega}, \quad (10)$$

with

$$S(T) = [\int_0^\infty a^2 f_1(a/T)da] f_2(T), \tag{11}$$

where the conditional PDF of wave amplitude $f_1(a/T)$ is given by (6) and (7), the PDF of wave period $f_2(T)$ is given by (4). Substituting this relation together with (4), (6), (7) and (8) into (10) and (11), we obtain

$$S(\omega) = \frac{(2n-1)^{n/2}}{\Gamma(\frac{n}{2})2^{n-3}} A_m^2 \frac{\omega_m^{2n+2r}}{\omega^{2n+2r+1}} exp[-\frac{(2n-1)\omega_m^4}{4\omega^4}]. \tag{12}$$

From $\frac{dS(\omega)}{d\omega}|_{\omega=\omega_p} = 0$, we find

$$\omega_p = (\frac{2n-1}{2n+2r+1})^{1/4}\omega_m. \tag{13}$$

where ω_m is the most-probable angular frequency. Substituting (13) into (12), we have

$$S(\omega) = \alpha(\frac{\omega_p}{\omega})^{2n+2r-4} \frac{g^2}{\omega^5} exp[-\frac{(2n+2r+1)}{4}\frac{\omega_p^4}{\omega^4}], \tag{14}$$

with the spectral coefficient

$$\alpha = \frac{(2n-1)^{(2-r)/2}(2n+2r+1)^{(n+r-2)/2}}{\Gamma(\frac{n}{2})2^{n-3}}(A_m k_m)^4, \tag{15}$$

where $k_m = \omega_m^2/g$ is the most-probable wavenumber.

b. Spectral width

The spectral width is defined as

$$\epsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}, \tag{16}$$

where m_i is the i-th moment of wave spectrum

$$m_i = \int_0^\infty \omega^i S(\omega) d\omega. \tag{17}$$

Substituting (16) and (17) into (14) and (15), we obtain

$$\epsilon = \sqrt{1 - \frac{\Gamma^2(\frac{n+r-1}{2})}{\Gamma(\frac{n+r}{2})\Gamma(\frac{n+r-2}{2})}}, \tag{18}$$

This formula represents the relationship between ϵ and $(n+r)$ for the derived spectrum. Equation (18) suggests that the spectral width ϵ is determined by both the spectral width-related parameter n and the correlation-related parameter r.

It is noted that the derived model (14) of equilibrium spectrum does not only represent the wave spectrum in the range of equilibrium range, but also can approximately to describe the wave spectrum in the whole range of gravity waves. For example, for $n+r = 3.0$, the derived spectrum (14) is proportional to the empirical model of Darbyshire (see Bretschneider 1959). For $n+r = 2.5$, the derived spectrum (14) is just the Neumann spectrum (Neumann 1952). For $n+r = 2$, the derived spectrum (14) is just the P-M spectrum (Pierson and Moscowitz, 1964) and proportional to the JONSWAP spectrum (Hasselmann et al. 1973). For $n+r = 1.5$, the derived spectrum (14) is proportional to the model (3) observed by Donelan et al.(1985). When $n+r < 2$, the spectral width $\epsilon \to \infty$, which means the spectral width defined by (18) doesn't exist. Only for a narrow subrange, the value of $n+r$ may be less than 2. For the whole equilibrium range, the average value of $n+r$ should be larger than 2. The model of Donelan et al. (1985) is proposed for a narrow range due to an unknown mechanism. This model cannot be extended to the whole equilibrium range, because the mean-square slope calculated from this model is much larger than the observations.

c. Spectral coefficient

Other than some previous models, the derived spectrum (14) has a finite fourth moment, hence a limited mean-square slope can be derived from (14) using

$$\sigma^2 = \int_0^\infty k^2 S(|\vec{k}|)kdk = \int_0^\infty \frac{\omega^4}{g^2} S(\omega) d\omega. \tag{19}$$

Substituting (14) into (19), we have

$$\alpha = 4\sigma^2(\frac{2n+2r+1}{4})^{\frac{n+r-2}{2}} / \Gamma(\frac{n+r-2}{2}), \tag{20}$$

where α is the spectral coefficient. For fully-developed waves, the mean-square slope, given by Liu et al. (1998), is

$$\sigma_f^2 = 0.0103 + 0.0092 \, ln \, U_{10}. \tag{21}$$

Substituting (9), (21) into (20), we obtain

$$\alpha_f = \frac{\sigma_f^2}{1.81}. \tag{22}$$

Substituting (9) and (22) into (14), the frequency spectrum in the equilibrium range for fully-developed waves can be expressed as

$$S_f(\omega) = \frac{\sigma_f^2}{1.81}(\frac{\omega_0}{\omega})^{0.5}\frac{g^2}{\omega^5} exp(-\frac{5.5}{4}\frac{\omega_0^4}{\omega^4}), \tag{23}$$

where ω_0 is the angular frequency at spectral peak for fully-developed waves, $\omega_0 = g/1.2U_{10}$. The corresponding uni-directional wavenumber spectrum in the equilibrium range is

$$S_f(k) = \beta_f k^{-4}, \tag{24}$$

with the spectral coefficient β_f for fully-developed stage

$$\beta_f = \frac{\sigma_f^2}{3.62}(\frac{k_0}{k})^{0.25} exp(-\frac{5.5}{4}\frac{k_0^2}{k^2}), \tag{25}$$

where k_0 is the wavenumber at spectral peak for fully-developed waves, $k_0 = \omega_0^2/g = g/(1.2U_{10})^2$.

Table 1: Spectral coefficient β_f

U_{10}	*wavenumber*				
	$2k_0$	$4k_0$	$6k_0$	$8k_0$	$10k_0$
3m/s	.00336	.00366	.00347	.00328	.00313
10m/s	.00519	.00564	.00535	.00506	.00482
17m/s	.00599	.00652	.00618	.00585	.00557
24m/s	.00651	.00709	.00672	.00636	.00606

Table 1 gives the values of spectral coefficient β_f in the range from k_0 to $10k_0$ for $U_{10} = 3, 10, 17$ and $24\ m/s$. These data are obtained from (25) and (21). The mean value of spectral coefficient β_f shown in Table 1 is 0.00529; the standard deviation is 0.00124. This value is very close to the saturation constant of 0.00615 ± 0.000615, obtained by Phillips (1977) based on previous investigations. The formulas (24) and (25) show a relationship of $k^{-4.25}$. Fig. 1 gives the sum of the equilibrium spectrum (24) with (25) and the gravity-capillary wave spectrum of Liu (1996). The sum of two spectra shows a relationship of approximate k^{-4}.

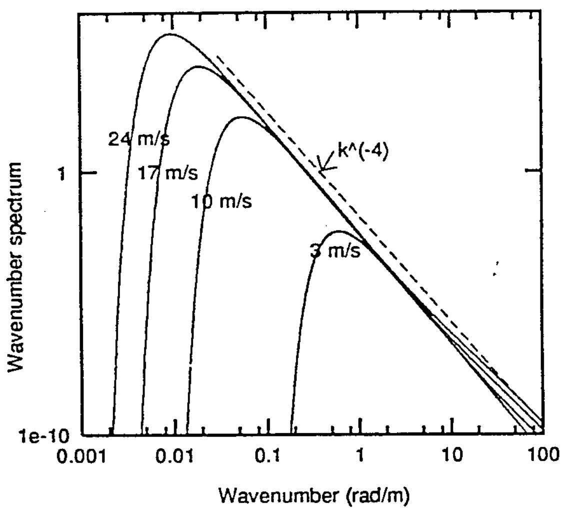

Figure 1: The sum of the equilibrium spectrum, given by (24) and (25), and the unidirectional wavenumber spectrum of wind-induced gravity-capillary waves, given by Liu (1996).

4 WAVE AGE

Substituting (20) into (14), we have

$$S(\omega) = \alpha' g^2 \omega^{-5}, \tag{26}$$

with

$$\alpha' = \frac{4\sigma^2 (\frac{2n+2r+1}{4})^{(n+r-2)/2}}{\Gamma(\frac{n+r-2}{2})} (\frac{\omega_p}{\omega})^{2n+2r-4} exp[-\frac{2n+2r+1}{4}\frac{\omega_p^4}{\omega^4}]. \tag{27}$$

The relationship between the mean-square slope σ^2 and the wave age, and the relationship between $(n+r)$ and the wave age will be determined, based on the spectral property of derived model and the observations of Donelan et al. (1985) and Perrie and Toulany (1990).

We use $\frac{\omega_0}{\omega_p}$ to represent the wave age, $\frac{\omega_p}{\omega_0}$ to represent the inverse wave age. Here ω_0 is the angular frequency at the spectral peak of the fully-developed stage, $\omega_0 = g/1.2U_{10}$, ω_p is the angular frequency at the spectral peak for the concerned waves. The inverse wave age can be defined as either $\frac{\omega_p}{\omega_0}$, or $\frac{U_{10}}{C_p}$, where C_p is the phase speed of waves at the spectral peak, or $\frac{U_{10}}{C_p} cos\theta$ (Donelan et al. 1985), where θ represents the wind direction.

The empirical model (3) of Donelan et al. (1985) shows that the spectral coefficient is proportional to the power of inverse wave age $(\frac{U_{10}}{C_p})^{0.55}$ or $(\frac{\omega_p}{\omega_0})^{0.55}$. So, we can assume

$$\sigma^2 = (\frac{\omega_p}{\omega_0})^{\nu} \sigma_f^2, \tag{28}$$

where σ^2 is the mean-square slope in (27), σ_f^2 is the mean-square slope in fully-developed stage, presented by (21).

The total energy of gravity waves E, also the zeroth moment of spectrum, is

$$E = \int_0^\infty S(\omega) d\omega. \tag{29}$$

Substituting (26) and (27) into (29), we obtain

$$E = \sigma^2 g^2 \omega_p^{-4} (\frac{2n+2r-4}{2n+2r+1}). \tag{30}$$

The dimensionless energy $\bar{E}$ and the dimensionless angular frequency at spectral peak $\bar{\omega}_p$ are defined as (Hasselmann et al. 1973)

$$\bar{E} = \frac{g^2 E}{U_{10}^4}, \tag{31}$$

and

$$\bar{\omega}_p = \frac{1.2\omega_p U_{10}}{g} = \frac{\omega_p}{\omega_0}. \tag{32}$$

Substituting (31) and (32) into (30), we have

$$\frac{2n+2r-4}{2n+2r-1} = \frac{\bar{E}\bar{\omega}_p^4}{\sigma^2}. \tag{33}$$

The relationship between $\bar{E}$ and $\bar{\omega}_p$ obtained by Perrie and Toulany (1990) is very close to that obtained by Donelan et al.(1985). Both formulas are based on the observations. From Donelan et al(1985)

$$\bar{E} \propto (\bar{\omega}_p)^{-3.3}. \tag{34}$$

Based on (28), (33) and (34), we can determine that

$$n + r = 2.5exp[\frac{1}{10.5}(\frac{\omega_p}{\omega_0})^{0.7-\nu}] - 0.5, \tag{35}$$

which denotes the relationship between $(n+r)$ and the inverse wave age $\frac{\omega_p}{\omega_0}$.

Substituting (35), (28) and (21) into (27), we can obtain the relationship between the spectral level α' and the inverse wave age $\frac{\omega_p}{\omega_0}$. Comparing α' with the spectral level observed by Donelan et al. (1985), we found that they fit well with each other when the value of ν is from 0 to 0.55. Combining the observations of spectral curve slopes (Imasato 1976), the value of ν is determined initially as 0.25.

The sum of spectral width-related parameter n and correlation-related parameter r is a function of the inverse wave age as given in (35). The mean-square slope is a function of the inverse wave age as shown in (28) and wind speed as showns in (21). So, the spectral level α' in (27) is a function of the inverse wave age and the wind speed. Approximately, we have

$$\sigma_f^2 = 0.0103 + 0.0092\, ln\, U_{10} \approx 0.0145 U_{10}^{0.32}. \qquad (36)$$

From (27), (28), (35) and (36) with $\nu = 0.25$, the proportional relationship on spectral level α' in (27) can be expressed as

$$\alpha' \propto (\frac{\omega_p}{\omega_0})^{0.55} U_{10}^{0.32}, \qquad (37)$$

which shows that the spectral level is a stronger function of the inverse wave age and a weaker function of the wind speed.

Substituting (28) and (35) into (33), the relationship between the dimensionless energy $\bar{E}$ and the inverse wave age $\frac{\omega_p}{\omega_0}$ is derived as

$$\bar{E} = (0.0103 + 0.0092\, lnU_{10})(\frac{\omega_p}{\omega_0})^{\nu-4}[1 - exp(-\frac{1}{10.5}\frac{\omega_p^{0.7-\nu}}{\omega_0^{0.7-\nu}})]. \qquad (38)$$

When $\nu = 0.25$, the two curves for $U_{10} = 3\ m/s$ and $24\ m/s$, respectively, are calculated from (38) and shown in Fig. 2 as solid lines. The empirical relations observed by Donelan et al. (1985) and by Perrie and Toulany (1990) are shown in Fig. 2 as diamonds and pluses.

The same order of the dimensionless energy between the derived model and the empirical relations reflects the accuracy of the proposed spectrum.

5 DEVELOPMENT OF WIND WAVES

The derived equilibrium spectrum can be described by (26) with (27). Although this model does no include the peak enhancement factor, it can also approximately describe the spectrum of gravity waves.

Fig. 3 shows the development process of gravity waves given by (26), (27), (28), (21) and (35) with $\nu = 0.25$ for $U_{10} = 10\ m/s$. When the wave age increases (the inverse wave age decreases), the position of spectral peak is moving from higher frequency to lower frequency, and the "overshooting" appears in the equilibrium range. Meanwhile, the corresponding spectral width ϵ calculated from

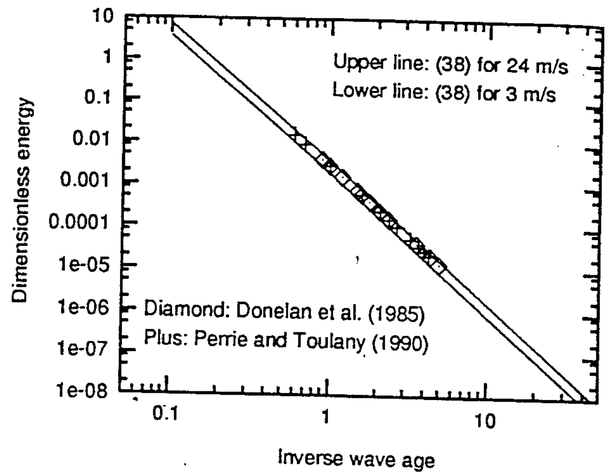

Figure 2: Dimensionless wave energy versus inverse wave age: Lines are given by (38) for $U_{10} = 24\ m/s$ and $3\ m/s$, respectively; diamonds represent the dimensionless wave energy of Donelan et al. (1985) and pluses represent the dimensionless wave energy of Perrie and Toulany (1990).

(18) is shown in the figure. The older the equilibrium spectrum, the larger is the spectral width. Younger waves have a narrower spectral width and a more steeper curve of the equilibrium spectrum. For example, In Fig. 3, the waves with an inverse wave age of 2.0 have a spectral width of 0.80 and a spectral relationship of $\omega^{-5.70}$; the waves with an inverse wave age of 1.0 have a spectral width of 0.84 and a relationship of $\omega^{-5.50}$; the waves with an inverse wave age of 0.5 have a spectral width of 0.88 and a relationship of $\omega^{-5.36}$; the waves with an inverse wave age of 0.1 have a spectral width of 0.94 and a relationship of $\omega^{-5.17}$. The frequency spectra with slopes from -5 to -6 have been observed previously (Imasato 1976).

6 SUMMARY

The equilibrium spectrum of wind waves is derived based on the joint-PDF of wave amplitude and wave period as

$$S(\omega) = 4\sigma_f^2 N(\frac{\omega_p}{\omega_0})^{\nu}(\frac{\omega_p}{\omega})^{2x-4} g^2 \omega^{-5} exp(-\frac{2x+1}{4}\frac{\omega_p^4}{\omega^4}), \qquad (39)$$

with

$$N = (\frac{2x+1}{4})^{(x-2)/2} / \Gamma(\frac{x-2}{2}), \qquad (40)$$

where $x = n + r$, which is related to the inverse wave age as shown in (35). Here "n" represents the spectral width-related parameter, "r" represents the correlation-related parameter. The related parameters of derived spectrum are determined based on the empirical relations. Therefore, this model is keeping with the observations. In (39), σ_f^2, as shown in (21), denotes the mean-square slope for fully-developed waves.

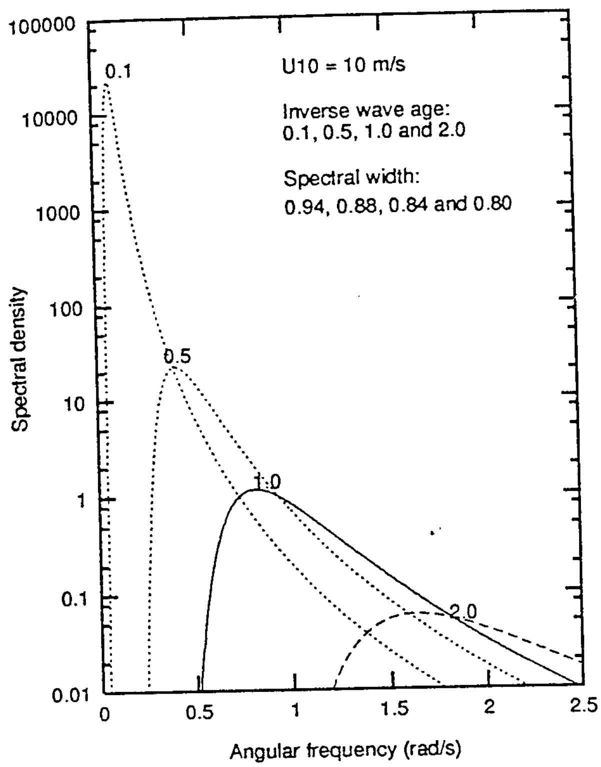

Figure 3: Spectral density (m^2s) versus angular frequency (rad/m): The four spectral curves describe the development of equilibrium spectra of wind waves for $U_{10} = 10\ m/s$. The four spectral curves are obtained from (26), (27), (28) and (35) with $\nu = 0.25$. These curves correspond to different inverse wave age and spectral width.

The derived model can explain the variability of the spectral level and the slope of spectral curve. The variability of spectral curve slope is related to the spectral width; the spectral width is controlled by the wave age. The smaller the wave age, the narrower the spectral width, and the larger the spectral curve slope. The variability of spectral level is due to both wind speed U_{10} and wave age $\frac{\omega_0}{\omega_p}$. The relationship between the spectral level and the mean-square slope is found from the fourth moment of spectrum. Furthermore, the relationship between the mean-square slope and the wind speed was found by Liu et al. (1998). The relationship between the spectral level and the inverse wave age is derived based on the observations of Donelan et al. (1985) and the spectral property on the zeroth moment and the fourth moment. The determination of power factor ν has not been solved completely, although an initial value of 0.25 is given. The relationship between the mean-square slope and the wave age can be used to explain the uncertainties of microwave radar measurements.

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